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Non(anti)commutative $\mathcal{N} = 2$ Supersymmetric Gauge Theory from Superstrings in the Graviphoton Background

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Abstract

We study open string amplitudes with the D3-branes in type IIB superstring theory compactified on $\mathbf{C}^2/\mathbf{Z}_2$. We introduce constant graviphoton background along the branes and calculate disk amplitudes using the NSR formalism. We take the zero slope limit and investigate the effective Lagrangian on the D3-branes deformed by the graviphoton background. We find that the deformed Lagrangian agrees with that of $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory defined in non(anti)commutative $\mathcal{N} = 1$ superspace by choosing appropriate graviphoton background. It is also shown that abelian gauge theory defined in $\mathcal{N} = 2$ harmonic superspace with specific non-singlet deformation is consistent with the deformed theory.

1 Introduction

It is known that the graviphoton effects play an important role for studying non-perturbative properties in superstring theory and supersymmetric gauge theory. The low energy dynamics of D-branes in the superstrings compactified on the Calabi-Yau manifold with the constant graviphoton background is shown to become supersymmetric gauge theories on non(anti)commutative superspace [1, 2, 3]. It is shown that the effective theory becomes supersymmetric Yang-Mills theory on $\mathcal{N} = 1/2$ superspace, which was constructed by Seiberg [4]. This theory is defined in $\mathcal{N} = 1$ Euclidean superspace by introducing non-anticommutativity for supercoordinates θ^α satisfying the Clifford algebra $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}$ [5, 6]. This theory is also considered as the low energy effective theory on the D3-branes of type IIB superstring theory compactified on $\mathbf{R}^6/\mathbf{Z}_2 \times \mathbf{Z}_2$ with constant graviphoton background [7].

Non(anti)commutative $\mathcal{N} = 1/2$ superspace can be generalized to extended superspace. Non(anti)commutative harmonic superspace [8] provides particularly an efficient tool for investigating the deformed Lagrangian and their symmetries at the off-shell level. The $\mathcal{N} = 2$ supersymmetric gauge theory on the non(anti)commutative harmonic superspace has been studied in [9, 10, 11, 12, 13], where one can introduce various types of deformation parameters by $\{\theta^{i\alpha}, \theta^{j\beta}\} = C^{\alpha\beta ij}$. Here $\theta^{i\alpha}$ is the supercoordinate labeled by $SU(2)_R$ R-symmetry index $i = 1, 2$.

The purpose of the present paper is to study the graviphoton effects in $\mathcal{N} = 2$ supersymmetric gauge theory, which can be obtained as the low-energy effective theory of the D3-brane in type IIB superstring theory. We will consider the (fractional) D3-branes in type IIB superstring theory compactified on the orbifold $\mathbf{C}^2/\mathbf{Z}_2$ [14]. We introduce constant graviphoton backgrounds along the branes and calculate disk amplitudes which remain nonzero in the zero slope limit. Here we will use the NSR formalism to introduce the graviphoton vertex operator in the closed string R-R sector. We construct the effective Lagrangian deformed by the graviphoton background. The constant graviphoton field strength $\mathcal{F}^{\alpha\beta ij}$ characterizes the deformation structure of the $\mathcal{N} = 2$ supersymmetric gauge theory on the branes.

There arise some non-trivial problems to compare two parameters $\mathcal{F}^{\alpha\beta ij}$ and $C^{\alpha\beta ij}$.

One is the choice of the scaling limit $(2\pi\alpha')^n\mathcal{F} = C =$ fixed for some n in the zero slope limit $\alpha' \to 0$. Here we take $\mathcal{F}$ such that it has mass dimension two. In this work we will fix $n = 3/2$ such that C becomes deformation parameters of non(anti)commutative superspace. Another point is the tensor structure of the graviphoton background. In the case of superstrings, spinor indices α, β and R-symmetry indices i, j are independent. But in the harmonic superspace formalism the deformation parameter $C^{ij}_{\alpha\beta}$ obeys symmetry $C^{ij}_{\alpha\beta} = C^{ji}_{\beta\alpha}$. This suggests that the graviphoton background $\mathcal{F}^{\alpha\beta ij}$ describes more general deformation of $\mathcal{N} = 2$ theory. We can classify the graviphoton background into four types $\mathcal{F}^{[\alpha\beta][ij]}$, $\mathcal{F}^{(\alpha\beta)[ij]}$, $\mathcal{F}^{[\alpha\beta](ij)}$, $\mathcal{F}^{(\alpha\beta)(ij)}$. Here the (square) bracket means (anti)symmetrization.

In this paper, we will study the $\mathcal{F}^{(\alpha\beta)(ij)}$ type deformation in detail. We will show that in the graviphoton background of type $\mathcal{F}^{(\alpha\beta)(ij)}$, the deformed Lagrangian includes that of $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory defined in $\mathcal{N} = 1/2$ superspace [15]. For the singlet type deformation $C^{ij}_{\alpha\beta} = C_s\epsilon^{ij}\epsilon_{\alpha\beta}$ [9], it is pointed that the deformed theory can be obtained from the constant R-R scalar background [8]. This deformation would correspond to the $\mathcal{F}^{[\alpha\beta][ij]}$ type deformation. However, for other types of graviphoton background $\mathcal{F}^{[\alpha\beta](ij)}$ and $\mathcal{F}^{(\alpha\beta)[ij]}$, they do not correspond to the deformed theory obtained from the non(anti)commutative harmonic superspace due to the difference of the tensor structures of indices.

Recently, Billó et. al. [14] studied the low-energy effective action in particular type constant graviphoton background and pointed its relation to the Ω-background which has been applied to obtain the exact prepotential formula [16]. They use the deformation of type $\mathcal{F}^{(\alpha\beta)[ij]}$ and different scaling $(2\pi\alpha')^{\frac{1}{2}}\mathcal{F}^{(\alpha\beta)[ij]} =$ fixed.

This paper is organized as follows: In section 2, we review type IIB superstrings on $\mathbf{C}^2/\mathbf{Z}_2$ using NSR formalism and construct $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in terms of the fractional D3-branes located at the singular point in the orbifold $\mathbf{C}^2/\mathbf{Z}_2$. We introduce auxiliary field vertex operators to simplify calculations of disk amplitudes. In section 3, we calculate the disk amplitudes with insertion of one graviphoton vertex operator. We focus on the $\mathcal{F}^{(\alpha\beta)(ij)}$ type background. In the case that only $\mathcal{F}^{(\alpha\beta)(11)}$ is non-zero, the deformed Lagrangian is shown to precisely agrees with the one that is constructed in $\mathcal{N} = 1$ non(anti)commutative superspace. We also show that by restricting to the abelian case, the deformed Lagrangian corresponds to the one which is defined in the non-singletly deformed harmonic superspace $\{\theta^{i\alpha}, \theta^{j\beta}\} = C^{\alpha\beta} b^{ij}$ with $b^{ij} b_{ij} = 0$. In section 4, we present our conclusions and discuss the possibility of new type of deformed $\mathcal{N} = 2$ gauge theory, that is obtained from the open superstring amplitudes. In appendix A, we summarize possible one graviphoton disk amplitudes which remain nonzero in the zero slope limit. In appendix B, we present some details for the effective rules in computing disk amplitudes including spin operators.

2 Type IIB superstrings on $\mathbf{C}^2/\mathbf{Z}_2$ and D3-branes

In this section we review the construction of the $\mathcal{N} = 2$ supersymmetric gauge theory with gauge group $U(N)$ by a stack of fractional D3-branes in type IIB superstring theory compactified on $\mathbf{C}^2/\mathbf{Z}^2$. We will use the NSR formalism.

2.1 Type IIB on $\mathbf{C}^2/\mathbf{Z}_2$

We begin with reviewing type II superstring theory in ten dimensions. Let $X^m(z, \bar{z})$, $\psi^m(z)$ and $\tilde{\psi}^m(\bar{z})$ $(m = 1, \cdots, 10)$ be free bosons and fermions with worldsheet coordinates $(z, \bar{z})$. Here we will take the Euclidean signature and their operator product expansions (OPEs) are given by $X^m(z)X^n(w) \sim -\delta^{mn} \ln(z - w)$ and $\psi^m(z)\psi^n(w) \sim \delta_{mn}/(z - w)$. Fermionic ghost system (b, c) with conformal weight $(2, -1)$ and bosonic ghost system (β, γ) with weight $(3/2, -1/2)$ are also introduced. The worldsheet fermions $\psi^m(z)$ are bosonized in terms of free bosons $\phi^a(z)$ $(a = 1, \cdots, 5)$ by

$$f^{\pm e_a}(z) \equiv \frac{1}{\sqrt{2}}(\psi^{2a-1} \mp i\psi^{2a}) =: e^{\phi^a}(z) : c_{e^a}. \tag{1}$$

Here $\phi^a(z)$ satisfy the OPE $\phi^a(z)\phi^b(w) \sim \delta^{ab} \ln(z-w)$ and the vectors e_a are orthonormal basis in the $SO(10)$ weight lattice space and c_{e^a} is a cocycle factor [17]. The bosonic ghost is also bosonized [18]: $\beta = \partial\xi e^{-\phi}$, $\gamma = e^{\phi}\eta$ with OPE $\phi(z)\phi(w) \sim -\ln(z - w)$. We will omit normal ordering symbol : : sometimes. In order to describe the R-sector, we need to introduce spin fields $S^\lambda(z) = e^{\lambda\phi}(z)c_\lambda$, where $\phi = \phi^a e_a$ and $\lambda = \frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5)$. λ belongs to the spinor representation of $SO(10)$. c_λ is a cocycle factor. In type IIB theory, after the GSO projection, we have spinor fields which have odd number of minus signs in λ, for both left and right movers.

We compactify the theory on $\mathbf{C} \times \mathbf{C}^2/\mathbf{Z}_2$ with internal coordinates $(x^5, \cdots, x^{10})$ and put the D3-branes with world volume in (x^1, x^2, x^3, x^4) directions. We introduce complex string coordinates and worldsheet fermions:

$$
\begin{aligned}
Z &= \frac{1}{\sqrt{2}}(X^5 + iX^6), \quad \Psi = \frac{1}{\sqrt{2}}(\psi^5 + i\psi^6), \\
Z^1 &= \frac{1}{\sqrt{2}}(X^7 + iX^8), \quad \Psi^1 = \frac{1}{\sqrt{2}}(\psi^7 + i\psi^8), \\
Z^2 &= \frac{1}{\sqrt{2}}(X^9 + iX^{10}), \quad \Psi^2 = \frac{1}{\sqrt{2}}(\psi^9 + i\psi^{10}).
\end{aligned} \tag{2}
$$

The $\mathbf{Z}_2$ action g acts on string coordinates as $(Z, Z_1, Z_2) \to (Z, -Z_1, -Z_2)$. For spinor states, g acts as $+\pi$ rotation on the $7-8$ and $-\pi$ rotation on the $9-10$ plane. Namely for a spin state $|\lambda_3, \lambda_4, \lambda_5\rangle$, g acts as $1 \otimes i\sigma_3 \otimes (-i\sigma_3)$, which breaks the $SO(6)$ spin symmetry into $SO(2) \times SU(2)$. $\mathbf{Z}_2$ invariant states are made of

$$
\left| \frac{\epsilon}{2}, \pm\frac{1}{2}, \pm\frac{1}{2} \right\rangle, \quad \epsilon = \pm 1.
$$

Ten-dimensional spinor field S^λ can be decomposed into $SO(4) \times SO(2) \times SU(2)$ under the orbifold projection:

$$
S^\lambda \to (S^\alpha S^{(-)} S^i, S^{\dot\alpha} S^{(+)} S^i) \tag{3}
$$

where S^α and $S^{\dot\alpha}$ $(\alpha, \dot\alpha = 1, 2)$ spinors are four-dimensional spinors with weights $\pm\frac{1}{2}(e_1 + e_2)$ and $\pm\frac{1}{2}(e_1 - e_2)$, respectively. We will follow the conventions of [19]. The upper and lower four-dimensional spinor indices are related by the anti-symmetric tensor $\varepsilon^{\alpha\beta}$. $S^{(\pm)} = e^{\pm\frac{1}{2}\phi_3}$ and S^i denote the internal spin fields. The S^i has weight $\pm\frac{1}{2}(e_4 + e_5)$. Similarly to the four-dimensional spinors, the internal spin indices i are raised and lowered by ε^{ij}.

When N D3-branes are located at the orbifold fixed point, the massless states describe $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory. The $\mathcal{N} = 2$ vector multiplet consists of gauge bosons A_μ, two gauginos $\Lambda^{\alpha i}$ $(i = 1, 2)$ and complex scalars φ, which belong to the adjoint representation of the gauge group.

We denote the vertex operator for a massless field X in the q-picture by $V_X^{(q)}$. For bosonic fields in the $\mathcal{N}=2$ vector multiplet, they are given by

$$\begin{aligned}
V_A^{(-1)} &= (2\pi\alpha')^{\frac{1}{2}} A^\mu(p) \frac{1}{\sqrt{2}} \psi_\mu e^{-\phi} e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V_\varphi^{(-1)} &= (2\pi\alpha')^{\frac{1}{2}} \varphi(p) \frac{1}{\sqrt{2}} \Psi e^{-\phi} e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V_{\bar{\varphi}}^{(-1)} &= (2\pi\alpha')^{\frac{1}{2}} \bar{\varphi}(p) \frac{1}{\sqrt{2}} \bar{\Psi} e^{-\phi} e^{i\sqrt{2\pi\alpha'}p\cdot X},
\end{aligned} \tag{4}$$

where p^μ is the four-momentum. For calculations of scattering amplitudes, we need vertex operators in 0-picture. These are given by

$$\begin{aligned}
V_A^{(0)} &= 2i(2\pi\alpha')^{\frac{1}{2}} A^\mu(p) \left(\partial X^\mu + i(2\pi\alpha')^{\frac{1}{2}} p\cdot\psi\psi^\mu\right) e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V_\varphi^{(0)} &= 2i(2\pi\alpha')^{\frac{1}{2}} \varphi(p) \left(\partial Z + i(2\pi\alpha')^{\frac{1}{2}} p\cdot\psi\Psi\right) e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V_{\bar{\varphi}}^{(0)} &= 2i(2\pi\alpha')^{\frac{1}{2}} \bar{\varphi}(p) \left(\partial \bar{Z} + i(2\pi\alpha')^{\frac{1}{2}} p\cdot\psi\bar{\Psi}\right) e^{i\sqrt{2\pi\alpha'}p\cdot X}.
\end{aligned} \tag{5}$$

For fermionic fields, they are constructed by using the spin fields:

$$\begin{aligned}
V_\Lambda^{(-1/2)} &= (2\pi\alpha')^{\frac{3}{4}} \Lambda^{\alpha i}(p) S_\alpha S^{(-)} S_i e^{-\frac{1}{2}\phi} e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V_{\bar{\Lambda}}^{(-1/2)} &= (2\pi\alpha')^{\frac{3}{4}} \bar{\Lambda}_{\dot{\alpha} i}(p) \bar{S}^{\dot{\alpha}} S^{(+)} S^i e^{-\frac{1}{2}\phi} e^{i\sqrt{2\pi\alpha'}p\cdot X}.
\end{aligned} \tag{6}$$

The prefactor of the vertex operators ensures that all the polarization has canonical dimension. Following [7], the Fourier transformation is taken with respect to the dimensionless momentum $k \equiv \sqrt{2\pi\alpha'}p$ so that the momentum polarization $A_\mu(p)$ has the same dimension of $A_\mu(x)$.

The graviphoton vertex operator belongs to the R-R sector and is expressed as

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z}) = (2\pi\alpha')\mathcal{F}^{\alpha\beta ij} e^{-\frac{1}{2}\phi} S_\alpha S^{(-)} S_i(z) e^{-\frac{1}{2}\phi} \tilde{S}_\beta \tilde{S}^{(-)} \tilde{S}_j(\bar{z}). \tag{7}$$

We normalized $\mathcal{F}^{\alpha\beta ij}$ such that it has canonical mass dimension $+2$.

2.2 Disk amplitudes

We now consider a disk amplitude such that open strings end on the D3-branes. The disk is realized as the upper half-plane whose boundary is real axis. The vertex operators for massless vector multiplets are inserted on the real axis and the graviphoton operators are in the upper-half plane. We apply the doubling trick where right-moving fields are located on the lower-half plane with the boundary condition:

$$S_\alpha S^{(-)} S_i(z) = \tilde{S}_\alpha \tilde{S}^{(-)} \tilde{S}_i(\bar{z})\Big|_{z=\bar{z}}. \tag{8}$$

The disk amplitudes can be calculated by replacing $\tilde{S}_\alpha \tilde{S}^{(-)} \tilde{S}_i(\bar{z})$ by $S_\alpha S^{(-)} S_i(\bar{z})$ in the correlator. The $n+2n_{\mathcal{F}}$-point disk amplitude for n vertex operators $V_{X_i}^{(q_i)}(y_i)$ and $n_{\mathcal{F}}$ graviphoton vertex operators $V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_j,\bar{z}_j)$ is given by

$$\langle\langle V_{X_1}^{(q_1)} \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})} \cdots \rangle\rangle = C_{D_2} \int \frac{\prod_{i=1}^{n} dy_i \prod_{j=1}^{n_{\mathcal{F}}} dz_j d\bar{z}_j}{dV_{CKG}} \langle V_{X_1}^{(q_1)}(y_1) \cdots V_{\mathcal{F}}^{(-\frac{1}{2},-\frac{1}{2})}(z_1,\bar{z}_1) \cdots \rangle. \tag{9}$$

Here C_{D_2} denotes the disk normalization factor, which is given by [20]

$$C_{D_2} = \frac{1}{2\pi^2(\alpha')^2} \frac{1}{k g_{\mathrm{YM}}^2} \tag{10}$$

where g_{YM} is the gauge coupling constant and k is a normalization of $U(N)$ generators T^a, $\mathrm{tr}(T^a T^b) = k\delta^{ab}$. dV_{CKG} is an $SL(2,\mathbf{R})$-invariant volume factor to fix three positions x_1, x_2 and x_3 among y_i, z_j,and $\bar{z}_j$'s:

$$dV_{CKG} = \frac{dx_1 dx_2 dx_3}{(x_1-x_2)(x_2-x_3)(x_3-x_1)}. \tag{11}$$

Note that in the disk amplitudes (9) the sum of the ϕ-charge in the bosonic ghost must be -2.

We need some correlation functions of ten-dimensional spin operators including bosonized ghosts such as $e^{-\frac{1}{2}\phi} S^\lambda(z)$, bosonized fermions $f^{\pm e_i}(z)$ and Lorentz generators $: f^{\pm e_i} f^{\pm e_j}(z) :$. Lorentz generators can be eliminated from the correlation functions by using the Ward identities (see Appendix B). The correlation functions are reduced to the ones of bosonized vertex operators of the form $e^{\tilde{\lambda}\cdot\tilde{\phi}}(z) c_{\tilde{\lambda}} = e^{\lambda\cdot\phi} e^{q\phi}(z) c_{\tilde{\lambda}}$. Here $\tilde{\lambda} = (\lambda, q)$ and $\tilde{\phi} = (\phi^a, \phi)$. The cocycle factor is given by $c_{\tilde{\lambda}} = \exp(\pi i \tilde{\lambda} M [\partial\tilde{\phi}]_0)$, where $[\partial\tilde{\phi}]_0$ denotes the zero mode of $\partial\tilde{\phi}$ and the 6×6 matrix M [17] is given by

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & -1 & -1 & 1 & 0 \end{pmatrix}. \tag{12}$$

Then the correlation functions are calculated as

$$\langle e^{\tilde{\lambda}_1\cdot\tilde{\phi}}(z_1)c_{\tilde{\lambda}_1}\cdots e^{\tilde{\lambda}_N\cdot\tilde{\phi}}(z_N)c_{\tilde{\lambda}_2}\rangle = \prod_{i<j}(z_i-z_j)^{\tilde{\lambda}_i\tilde{\lambda}_j}\exp(\pi i\tilde{\lambda}_i\cdot M\tilde{\lambda}_j)\delta_{\sum_i\tilde{\lambda}_i,(0,-2)}. \tag{13}$$

Here $\tilde{\lambda}_i\cdot\tilde{\lambda}_j = \lambda_i\cdot\lambda_j - q_iq_j$ for $\tilde{\lambda}_i = (\lambda_i, q_i)$. When we decompose the spin operators as in (3), we can obtain the "effective" rules for space-time and internal parts [21]. These rules are summarized in Appendix B.

2.3 $\mathcal{N}=2$ gauge theory and the auxiliary field method

The action of $\mathcal{N}=2$ supersymmetric Yang-Mills theory is given by

$$\begin{aligned} S^{\mathcal{N}=2}_{\mathrm{SYM}} &= \int d^4x\,\frac{1}{g^2_{\mathrm{YM}}}\frac{1}{k}\mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}-D_\mu\varphi D^\mu\bar{\varphi}-\frac{1}{2}\left[\varphi,\bar{\varphi}\right]^2\right.\\ &\qquad \left.-i\Lambda^i\sigma^\mu D_\mu\bar{\Lambda}_i-\frac{1}{\sqrt{2}}\Lambda^i\left[\bar{\varphi},\Lambda_i\right]-\frac{1}{\sqrt{2}}\bar{\Lambda}_i\left[\varphi,\bar{\Lambda}^i\right]\right), \end{aligned} \tag{14}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu-\partial_\nu A_\mu+i[A_\mu,A_\nu],\\ D_\mu\varphi &= \partial_\mu\varphi+i[A_\mu,\varphi], \end{aligned} \tag{15}$$

and $\tilde{F}_{\mu\nu}$ is the dual of $F_{\mu\nu}$. $\sigma_\mu = (i\tau^1, i\tau^2, i\tau^3, 1)$ and $\bar{\sigma}_\mu = (-i\tau^1,-i\tau^2,-i\tau^3,1)$ are Dirac matrices. Here τ^a $(a=1,2,3)$ denote the Pauli matrices. The gauge fields A_μ and other fields are expanded such as $A_\mu = A^a_\mu T^a$. In the action (14) we have eliminated auxiliary fields of the superfields. The action is derived by computing disk amplitudes with vertex operators attached on the boundary of the disk.

The auxiliary field method [7] (see also [21, 22]) is found to give an effective tool to simplify calculations because four-point amplitudes can be reduced to an three-point amplitudes which include an auxiliary field vertex operator. In [7], this method was applied to obtain non(anti)commutative $\mathcal{N}=1/2$ super Yang-Mills theory from the D3-brane in type IIB superstrings compactified on $\mathbf{C}^3/\mathbf{Z}_2\times\mathbf{Z}_2$. In this paper we generalize this method to the case of the $\mathcal{N}=2$ gauge theory.

In [7], it was shown that the quartic interactions of gauge fields can be written into the cubic type interactions by introducing the auxiliary self-dual tensor $H_{\mu\nu}$, which is also expressed in terms of 't Hooft eta symbol such as $H_{\mu\nu} = H^c \eta^c_{\mu\nu}$. The gauge field part $-\frac{1}{4g^2_{\rm YM}k}{\rm tr}(F^2_{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu})$ in the Lagrangian is equivalent to

$$-\frac{1}{g^2_{\rm YM}}\frac{1}{k}{\rm tr}\left(\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + i\partial_\mu A_\nu[A^\mu, A^\nu] + \frac{1}{2}H_c H^c + \frac{1}{2}H_c \eta^c_{\mu\nu}[A^\mu, A^\nu]\right). \tag{16}$$

In the $\mathcal{N} = 2$ case, the action (14) contains other quartic interactions which include scalar fields and gauge fields. We therefore introduce new auxiliary fields $H_{A\varphi_\mu}$, $H_{A\bar{\varphi}_\mu}$ and $H_{\varphi\bar{\varphi}}$. The Lagrangian is shown to be equal to

$$\begin{aligned}
\mathcal{L}^{\mathcal{N}=2}_{\rm SYM} \;=\;& -\frac{1}{g^2_{YM}}\frac{1}{k}{\rm tr}\Big[\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + i\partial_\mu A_\nu[A^\mu, A^\nu] + \frac{1}{2}H_c H^c + \frac{1}{2}H_c\eta^c_{\mu\nu}[A^\mu, A^\nu] \\
&+\partial_\mu\varphi\partial^\mu\bar{\varphi} + i\partial_\mu\varphi[A^\mu, \bar{\varphi}] + i[A_\mu, \varphi]\partial\bar{\varphi} - H_{A\varphi\mu}H^\mu_{A\bar{\varphi}} + iH_{A\varphi\mu}[A^\mu, \bar{\varphi}] + i[A_\mu, \varphi]H^\mu_{A\bar{\varphi}} \\
&+H^2_{\varphi\bar{\varphi}} + i\sqrt{2}H_{\varphi\bar{\varphi}}[\varphi, \bar{\varphi}] \\
&-i\Lambda^i\sigma^\mu D_\mu\bar{\Lambda}_i - \frac{1}{\sqrt{2}}\Lambda^i\left[\bar{\varphi}, \Lambda_i\right] - \frac{1}{\sqrt{2}}\bar{\Lambda}_i\left[\varphi, \bar{\Lambda}^i\right]\Big].
\end{aligned} \tag{17}$$

The auxiliary fields have relevant vertex operators in superstring theory. In [7], it is shown that the auxiliary fields $H_{\mu\nu}$ is associated to the vertex operator

$$V^{(0)}_H(y) = \frac{1}{2}(2\pi\alpha')H_{\mu\nu}(p)\psi^\nu\psi^\mu e^{i\sqrt{2\pi\alpha'}p\cdot X}(y) \tag{18}$$

in the 0-picture. In the $\mathcal{N} = 2$ case this vertex operator can be generalized to other auxiliary fields such as

$$\begin{aligned}
V^{(0)}_{H_{A\varphi}} &= 2i(2\pi\alpha')H_{A\varphi_\mu}\psi^\mu\Psi e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V^{(0)}_{H_{A\bar{\varphi}}} &= 2i(2\pi\alpha')H_{A\bar{\varphi}_\mu}\psi^\mu\bar{\Psi} e^{i\sqrt{2\pi\alpha'}p\cdot X}, \\
V^{(0)}_{H_{\varphi\bar{\varphi}}} &= -i\sqrt{2}(2\pi\alpha')H_{\varphi\bar{\varphi}}\Psi\bar{\Psi} e^{i\sqrt{2\pi\alpha'}p\cdot X}.
\end{aligned} \tag{19}$$

We now explain that all the interaction terms in the $\mathcal{N} = 2$ Lagrangian (17) can be derived from the disk amplitudes with vertex operators on the boundary. For example, the $H_{\mu\nu}[A^\mu, A^\nu]$ term in (17) is derived from the disk amplitude

$$\begin{aligned}
&\langle\!\langle V^{(0)}_H(p_1)V^{(-1)}_A(p_2)V^{(-1)}_A(p_3)\rangle\!\rangle \\
&\quad= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg^2_{\rm YM}}(2\pi\alpha')^2\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2 {\rm tr}\left[H_{\mu\nu}(p_1)A_\rho(p_2)A_\sigma(p_3)\right]\int\frac{\prod_j dy_j}{dV_{\rm CKG}} \\
&\qquad\times\langle e^{-\phi(y_2)}e^{-\phi(y_3)}\rangle\langle\psi^\nu\psi^\mu(y_1)\psi^\rho(y_2)\psi^\sigma(y_3)\rangle\left\langle\prod_{j=1}^3 e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle.
\end{aligned} \tag{20}$$

Here we have separated the correlator into the four-dimensional, internal and ghost parts. The ghost part can be evaluated by the Wick formula. The other parts are calculated by the effective rules in Appendix B. The X^μ correlator is given by

$$\left\langle \prod_{j=1}^{3} e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)} \right\rangle = (y_1-y_2)^{2\pi\alpha' p_1\cdot p_2}(y_1-y_3)^{2\pi\alpha' p_1\cdot p_3}(y_2-y_3)^{2\pi\alpha' p_2\cdot p_3}. \tag{21}$$

Since we consider only the massless states, we have $p_i \cdot p_j = 0$ for $i, j = 1\cdots 3$ and the contribution from the X correlator becomes trivial. Taking all together, the amplitude is

$$\langle\langle V_H^{(0)}(p_1)V_A^{(-1)}(p_2)V_A^{(-1)}(p_3)\rangle\rangle = \frac{1}{g_{\mathrm{YM}}^2 k}\mathrm{tr}\left[H_{\mu\nu}(p_1)A^\mu(p_2)A^\nu(p_3)\right]. \tag{22}$$

We note that the appropriate α' scaling appeared. After adding the other inequivalent color ordered amplitudes to the above one and taking the symmetric factor into account, we find the interaction term corresponding to the amplitude is

$$\mathcal{L} = -\frac{1}{2g_{\mathrm{YM}}^2}\frac{1}{k}\mathrm{tr}\left[H_{\mu\nu}(x)[A^\mu(x), A^\nu(x)]\right], \tag{23}$$

which is precisely the desired interaction in (17). The other interaction can be calculated by the same way and the results are

$$\begin{aligned}\left\langle\left\langle V_A^{(0)}(p_1)V_A^{(-1)}(p_2)V_A^{(-1)}(p_3)\right\rangle\right\rangle = -\frac{2}{kg_{\mathrm{YM}}^2}\mathrm{tr}\big[&A_\mu(p_1)p_2^\mu A_\rho(p_2)A_\sigma(p_3)\delta^{\rho\sigma}\\ &+p_{1\nu}A_\mu(p_1)A_\rho(p_2)A_\sigma(p_3)\delta^{\mu\rho}\delta^{\nu\sigma}\\ &-p_{1\nu}A_\mu(p_1)A_\rho(p_2)A_\sigma(p_3)\delta^{\mu\sigma}\delta^{\nu\rho}\big],\end{aligned} \tag{24}$$

$$\left\langle\left\langle V_{H_{\varphi\bar\varphi}}^{(0)}(p_1)V_\varphi^{(-1)}(p_2)V_{\bar\varphi}^{(-1)}(p_3)\right\rangle\right\rangle = \frac{i\sqrt{2}}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[H_{\varphi\bar\varphi}(p_1)\varphi(p_2)\bar\varphi(p_3)\right], \tag{25}$$

$$\left\langle\left\langle V_{H_{A\varphi}}^{(0)}(p_1)V_A^{(-1)}(p_2)V_{\bar\varphi}^{(-1)}(p_3)\right\rangle\right\rangle = \frac{2i}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[H_{A\varphi\mu}(p_1)A^\mu(p_2)\bar\varphi(p_3)\right], \tag{26}$$

$$\left\langle\left\langle V_\varphi^{(0)}(p_1)V_A^{(-1)}(p_2)V_{\bar\varphi}^{(-1)}(p_3)\right\rangle\right\rangle = -\frac{2}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[p_{1\mu}\varphi(p_1)A^\mu(p_2)\bar\varphi(p_3)\right], \tag{27}$$

$$\left\langle\left\langle V_\Lambda^{(-1/2)}(p_1)V_\Lambda^{(-1/2)}(p_2)V_{\bar\varphi}^{(0)}(p_3)\right\rangle\right\rangle = -\frac{\sqrt{2}}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[\Lambda^{\alpha i}(p_1)\Lambda_{\alpha i}(p_2)\bar\varphi(p_3)\right], \tag{28}$$

$$\left\langle\left\langle V_\Lambda^{(-1/2)}(p_1)V_A^{(-1)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)\right\rangle\right\rangle = \frac{1}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[\Lambda^{\alpha i}(p_1)(\sigma^\mu)_\alpha{}^{\dot\beta}A_\mu(p_2)\overline{\Lambda}_{\dot\beta j}(p_3)\right]. \tag{29}$$

Adding other inequivalent color ordered amplitudes and the phase shift of Λ, we find that all the cubic interactions in (17) are reproduced from these disk amplitudes.

3 Disk amplitudes in the constant graviphoton background

In this section, we will calculate the correction to the disk amplitudes due to the insertion of one graviphoton vertex operator.

3.1 The zero slope limit

We now examine the effect of the graviphoton vertex operator inserted in the disk. We will take the zero slope (field theory) limit $\alpha' \to 0$ at the final stage of the amplitudes calculation. The R-R graviphoton vertex operator in the disk amplitudes is written as

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar{z}){=}(2\pi\alpha')\mathcal{F}^{\alpha\beta ij}\left[S_\alpha(z)S^{(-)}(z)S_i(z)e^{-\frac{1}{2}\phi(z)}S_\beta(\bar{z})S^{(-)}(\bar{z})S_j(\bar{z})e^{-\frac{1}{2}\phi(\bar{z})}\right], \quad (30)$$

where we identify the left- and right-moving part.

We need to fix the scaling of the constant graviphoton background. In general we can take the limit such that

$$(2\pi\alpha')^n\mathcal{F}^{\alpha\beta ij} = C^{\alpha\beta ij} \quad (31)$$

is fixed for some n. For $n = 3/2$, the parameter $C^{\alpha\beta ij}$ has mass dimensions -1, which has the same dimension as the deformation parameters in the non(anti)commutative field theory.

We firstly explore which type of disk amplitudes remain nonzero in the zero slope limit. Let n_X be the number of vertex operators for a massless field X. Assuming that we can assign the appropriate picture number for each vertex operators, the amplitudes of the form $\langle\langle V_A^{n_A}\cdots V_{\bar{\varphi}}^{n_{\bar{\varphi}}}V_{\mathcal{F}}^{n_{\mathcal{F}}}\rangle\rangle$ scale as $(\alpha')^M$, where

$$M = -2 + \frac{1}{2}(n_A + n_\varphi + n_{\bar{\varphi}}) + \frac{3}{4}(n_\Lambda + n_{\bar{\Lambda}}) + (1-n)\, n_{\mathcal{F}}. \quad (32)$$

Here -2 comes from the normalization of the disk amplitudes. For $M \le 0$, the amplitudes remains nonzero in the zero-slope limit. Using the ϕ_3 charge conservation we get

$$-n_\varphi + n_{\bar{\varphi}} - \frac{1}{2}n_\Lambda + \frac{1}{2}n_{\bar{\Lambda}} - n_{\mathcal{F}} = 0. \quad (33)$$

Using (32) and (33), the condition $M \le 0$ becomes

$$\frac{1}{2}n_A + \left(n-\frac{1}{2}\right)n_\varphi + \left(\frac{3}{2}-n\right)n_{\bar{\varphi}} + \frac{1}{2}\left(n+\frac{1}{2}\right)n_\Lambda + \frac{1}{2}\left(\frac{5}{2}-n\right)n_{\bar{\Lambda}} \le 2. \quad (34)$$

We then can classify which type of amplitudes remain non-zero in the zero-slope limit. This analysis can be generalized to the amplitudes including auxiliary field vertex operators. Let n_Y^H be the number of vertex operators for auxiliary fields H_Y. Then the condition (34) becomes

$$\frac{1}{2}n_A+\left(n-\frac{1}{2}\right)n_\varphi+\left(\frac{3}{2}-n\right)n_{\bar\varphi}+\frac{1+2n}{4}n_\Lambda+\frac{5-2n}{4}n_{\bar\Lambda}+n^H_{AA}+n^H_{\varphi\bar\varphi}+nn^H_{A\varphi}+(2-n)n^H_{A\bar\varphi}\leq 2 \tag{35}$$

with the ϕ_3-charge conservation

$$-n_\varphi+n_{\bar\varphi}-\frac{1}{2}n_\Lambda+\frac{1}{2}n_{\bar\Lambda}-n^H_{A\varphi}+n^H_{A\bar\varphi}-n_{\mathcal{F}}=0. \tag{36}$$

We now consider the case $n=3/2$. In this case, the condition (35) becomes

$$\frac{1}{2}n_A+n_{\bar\varphi}+n_\Lambda+\frac{1}{2}n_{\bar\Lambda}+n^H_{AA}+n^H_{\varphi\bar\varphi}+\frac{3}{2}n^H_{A\varphi}+\frac{1}{2}n^H_{A\bar\varphi}\leq 2. \tag{37}$$

Without auxiliary fields, we find that 17 types amplitudes remain non-zero. For example, $A^4\bar\varphi^{n_\mathcal{F}}\mathcal{F}^{n_\mathcal{F}}$ type amplitudes remain non-vanishing in the zero slope limit for $n_\mathcal{F}\geq 0$. This infinite series type correction arises also in the case of non(anti)commutative harmonic superspace [8, 12]. Indeed, this systematic analysis of α' scaling is only a sufficient condition for the non-vanishing amplitudes in the field theory limit. In the string theory there is no guarantee that the amplitude is non-vanishing even though it has an appropriate α' scaling.

In this work we will consider the lowest order correction to the amplitude by one constant graviphoton vertex operator.

3.2 Disk amplitudes in the zero slope limit with fixed $(2\pi\alpha')^{\frac{3}{2}}\mathcal{F}$

We will examine possible structure of string amplitudes in the scaling limit with fixed $(2\pi\alpha')^{\frac{3}{2}}\mathcal{F}$. But it is necessary to see the explicit from of the correlator before the zero-slope limit is taken.

Focusing on the ϕ_3 charge, the graviphoton vertex operator contains two internal spin fields $S^{(-)}$. To cancel this ϕ_3 charge, one should insert one $\bar\varphi$ vertex or two $\overline{\Lambda}$ vertex operators. Thus the non-zero disk amplitudes that include one graviphoton vertex operators should be of the form

$$\langle\langle\cdots V_{\bar\varphi}V_{\mathcal{F}}\rangle\rangle,\qquad \langle\langle\cdots V_{\overline{\Lambda}}V_{\overline{\Lambda}}V_{\mathcal{F}}\rangle\rangle, \tag{38}$$

where remaining part is ϕ_3 neutral. Possible insertions are of the form

$$V_A,\ V_\varphi V_{\bar\varphi},\ V_\Lambda V_{\overline{\Lambda}},\ V_\varphi V_{\overline{\Lambda}} V_{\overline{\Lambda}},\ V_{\bar\varphi} V_\Lambda V_\Lambda. \tag{39}$$

Thus, the non-zero disk amplitudes for the one graviphoton vertex insertion have the structure of (38) with the insertion of the vertex operators in (39).

As mentioned in [23], when we have non-zero amplitude with a 0-picture vertex operator $V_X^{(0)}$ corresponding to the fields $X = (A_\mu, \varphi, \bar\varphi)$ (which produces the derivative $\partial_\mu X$), the amplitude in which $V_X^{(0)}$ is replaced by $V_{H_{AX}}^{(0)}$ is also non-zero. The combined amplitude $\langle\langle (V_X^{(0)} + V_{H_{AX}}^{(0)}) \cdots \rangle\rangle$ correspond to the gauge covariant derivative $D_\mu X$. Thus, whenever we have non-zero amplitude in the zero-slope limit which contains 0-ghost picture vertex operator $V_X^{(0)}$, we should consider the auxiliary field vertex operator $V_{H_{AX}}^{(0)}$ to obtain gauge covariant result.

We do not need to calculate the interaction $[A_\mu, A_\nu]^2$ which can be generated after the integration of the auxiliary field $H_{\mu\nu}$ in the Lagrangian, at the string level. Such interactions must be carefully extracted from those presented in the previous subsection.

We summarize all the possible vertex insertions that survives in the zero-slope limit in appendix A. Any other amplitudes containing one graviphoton and open string vertex operators vanish in the zero-slope limit or are not consistent with the auxiliary field Lagrangian.

3.3 Graviphoton effect

Before calculating the corrections explicitly, we will examine the tensor structure of $\mathcal{F}^{\alpha\beta ij}$. Since the space-time spinor indices α and the R-symmetry indices i are independent, we can classify the deformations as follows: $\mathcal{F}^{[\alpha\beta][ij]}$, $\mathcal{F}^{(\alpha\beta)[ij]}$, $\mathcal{F}^{[\alpha\beta](ij)}$ and $\mathcal{F}^{(\alpha\beta)(ij)}$. We call these as (S,S), (S,A), (A,S), (A,A) type respectively. The general background contains all of these types simultaneously. It may be better to investigate each type of deformation separately. In the following, we consider only the (S,S) type of the graviphoton background $\mathcal{F}^{\alpha\beta ij} = \mathcal{F}^{(\alpha\beta)(ij)}$. As we will see, this type of background corresponds to the graviphoton field strength that induces non(anti)commutative $\mathcal{N} = 1$ superspace $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}$ and the non-singletly deformed $\mathcal{N} = 2$ harmonic superspace $\{\theta^{i\alpha}, \theta^{j\beta}\} = C^{\alpha\beta} b^{ij}$. The vertex operator for the graviphoton field strength contains two internal spin fields S_i, S_j. These internal spin fields, when inserted on the disk without any other internal spin fields, generates anti-symmetric tensor ε_{ij} through the correlator $\langle S_i(z)S_j(w)\rangle \sim \varepsilon_{ij}$. When this anti-symmetric tensor is contracted with the graviphoton field strength $\mathcal{F}^{(\alpha\beta)(ij)}$, it gives vanishing amplitude. So, to obtain the non-vanishing amplitudes, at least one fermion vertex operator should be inserted in addition to the one graviphoton vertex operator. The cancellation condition of the ϕ_3-charge implies that the smallest number of fermion insertion is actually two. Let us examine all the possible amplitudes including two fermion vertex operators below.

- $\langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_{\bar\varphi} V_{\mathcal{F}}\rangle\!\rangle + \langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_{H_{A\bar\varphi}} V_{\mathcal{F}}\rangle\!\rangle$

The first example of the amplitudes is $\langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_{\bar\varphi} V_{\mathcal{F}}\rangle\!\rangle$. We should assign the picture number to each vertex operators adequately and evaluate the correlators. The amplitude is

$$\begin{aligned}
&\langle\!\langle V_\Lambda^{(-1/2)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{\bar\varphi}^{(0)}(p_3) V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle_{(\mathrm{S,S})} \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2\pi\alpha')^3(2i)\mathrm{tr}\left[\Lambda^{\gamma k}(p_1)\overline{\Lambda}_{\dot\delta l}(p_2)\bar\varphi(p_3)\right]\mathcal{F}^{(\alpha\beta)(ij)} \\
&\quad\times\int\frac{\prod_j dy_j}{dV_{\mathrm{CKG}}}\langle e^{-\frac{1}{2}\phi(y_1)}e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar z)}\rangle\langle S_k(y_1)S^l(y_2)S_i(z)S_j(\bar z)\rangle \\
&\quad\times i(2\pi\alpha')^{\frac{1}{2}}p_{3\mu}\langle S_\gamma(y_1)S^{\dot\delta}(y_2)\psi^\mu(y_3)S_\alpha(z)S_\beta(\bar z)\rangle \\
&\quad\times\langle S^{(-)}(y_1)S^{(+)}(y_2)\overline{\Psi}(y_3)S^{(-)}(z)S^{(-)}(\bar z)\rangle\left\langle\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle.
\end{aligned}\tag{40}$$

Here the symbol "(S,S)" means that we extract only the non-zero part from the correlator when it is contracted with the (S,S) type of the graviphoton field strength $\mathcal{F}^{(\alpha\beta)(ij)}$. After using the effective rules and also the massless condition, we see

$$\begin{aligned}
&\langle\!\langle V_\Lambda^{(-1/2)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{\bar\varphi}^{(0)}(p_3) V_{\mathcal{F}}^{(-1)}\rangle\!\rangle_{(\mathrm{S,S})} \\
&= -\frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2\pi\alpha')^{\frac{7}{2}}(2i^2)\frac{1}{\sqrt{2}}(\sigma^\mu)_\alpha{}^{\dot\delta}\varepsilon_{\gamma\beta}e^{-\frac{1}{4}\pi i}\cdot I\cdot\mathrm{tr}\left[\Lambda_{\beta j}(p_1)\overline{\Lambda}_{\dot\delta j}(p_2)p_{3\mu}\bar\varphi(p_3)\right]\mathcal{F}^{(\alpha\beta)(ij)}.
\end{aligned}\tag{41}$$

Here, the overall phase which comes from the cocycle factor and spin fields [17] is explicitly written. The $SL(2,\mathbf{R})$ invariance is used to fix the positions to $y_1\to\infty, z\to i, \bar z\to -i$

[7]. I is the world sheet integral and is evaluated as

$$I = \int_{-\infty}^{\infty} dy_2 \int_{-\infty}^{y_2} dy_3 \, \frac{(2i)^2}{(y_2^2+1)(y_3^2+1)} = (2i)^2 \frac{\pi^2}{2}. \tag{42}$$

After all, the resulting amplitude is

$$\begin{aligned}&\langle\!\langle V_\Lambda^{(-1/2)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{\bar{\varphi}}^{(0)}(p_3) V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle_{(\mathrm{S,S})} \\ &\quad = -\frac{2}{\sqrt{2}} \frac{1}{kg_{\mathrm{YM}}^2} \mathrm{tr} \left[\Lambda_{\alpha i}(p_1) \overline{\Lambda}_{\dot{\alpha} j}(p_2) (\sigma^\mu)_\beta{}^{\dot{\alpha}} i p_{3\mu} \bar{\varphi}(p_3) \right] C^{(\alpha\beta)(ij)},\end{aligned} \tag{43}$$

where we have defined $C^{(\alpha\beta)(ij)} \equiv -4\pi^2 e^{\frac{1}{4}\pi i} (2\pi\alpha')^{\frac{3}{2}} \mathcal{F}^{(\alpha\beta)(ij)}$. By adding another color ordered contribution, we find that the amplitude is reproduced by the following interaction:

$$\mathcal{L} = -\frac{1}{\sqrt{2}} \frac{1}{kg_{\mathrm{YM}}^2} \mathrm{tr} \left[C^{(\alpha\beta)(ij)} \left\{ \partial_\mu \bar{\varphi}(x), (\sigma^\mu)_{\alpha\dot{\alpha}} \overline{\Lambda}^{\dot{\alpha}}{}_i(x) \right\} \Lambda_{\beta j}(x) \right]. \tag{44}$$

This result contains derivative of the adjoint scalar which originates from the zero-ghost picture vertex operator $V_{\bar{\varphi}}^{(0)}$. As we noticed before, the auxiliary field amplitude $\langle\!\langle V_\Lambda V_{\overline{\Lambda}} V_{H_{A\bar{\varphi}}} V_{\mathcal{F}} \rangle\!\rangle$ also contributes. In a similar way, the auxiliary field contribution is evaluated to be

$$\begin{aligned}&\langle\!\langle V_\Lambda^{(-1/2)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{H_{A\bar{\varphi}}}^{(0)}(p_3) V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle_{(S,S)} \\ &\quad = \frac{2}{\sqrt{2}} \frac{1}{kg_{\mathrm{YM}}^2} \mathrm{tr} \left[\Lambda_{\alpha i}(p_1) \overline{\Lambda}_{\dot{\alpha} j}(p_2) (\sigma^\mu)_\beta{}^{\dot{\alpha}} H_{A\bar{\varphi}_\mu}(p_3) \right] C^{(\alpha\beta)(ij)}.\end{aligned} \tag{45}$$

After adding other inequivalent color order and multiplying the symmetric factor, we find the sum of the above two interactions is written as

$$\mathcal{L} = -\frac{1}{\sqrt{2}} \frac{1}{kg_{\mathrm{YM}}^2} \mathrm{tr} \left[C^{(\alpha\beta)(ij)} \left\{ \partial_\mu \bar{\varphi}(x) + H_{A\bar{\varphi}_\mu}(x), (\sigma^\mu)_{\alpha\dot{\alpha}} \overline{\Lambda}^{\dot{\alpha}}{}_i(x) \right\} \Lambda_{\beta j}(x) \right]. \tag{46}$$

- $\underline{\langle\!\langle V_A V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\!\rangle + \langle\!\langle V_H V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\!\rangle}$

The next possible amplitude that can survive is $\langle\!\langle V_{V_A} V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\!\rangle$, which is given by

$$\begin{aligned}&\langle\!\langle V_A^{(0)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{\overline{\Lambda}}^{(-1/2)}(p_3) V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle_{(\mathrm{S,S})} \\ &\quad = \frac{1}{2\pi^2\alpha'^2} \frac{1}{kg_{\mathrm{YM}}^2} (2i)(2\pi\alpha')^3 \mathrm{tr} \left[A_\mu(p_1) \overline{\Lambda}_{\dot{\alpha} k}(p_2) \overline{\Lambda}_{\dot{\beta} l}(p_3) \right] \mathcal{F}^{(\alpha\beta)(ij)} \int \frac{\prod_j dy_j}{dV_{\mathrm{CKG}}}\end{aligned}$$

$$
\begin{aligned}
&\times\langle e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(y_3)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle\langle S^k(y_2)S^l(y_3)S_i(z)S_j(\bar{z})\rangle\\
&\times\langle S^{(+)}(y_2)S^{(+)}(y_3)S^{(-)}(z)S^{(-)}(\bar{z})\rangle\\
&\times i(2\pi\alpha')^{\frac{1}{2}}p_{1\nu}\langle\psi^\nu\psi^\mu(y_1)S^{\dot{\alpha}}(y_1)S^{\dot{\beta}}(y_2)S_\alpha(z)S_\beta(\bar{z})\rangle\left\langle\prod_{j=1}^{3}e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle\\
=&\frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2i^2)(4\pi^2\alpha'^2)\cdot I\cdot\frac{1}{2}e^{-\frac{1}{4}\pi i}\mathrm{tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}p_{1\mu}A_\nu(p_1)\overline{\Lambda}_{\dot{\alpha}i}(p_2)\overline{\Lambda}_{\dot{\beta}i}(p_3)\right](2\pi\alpha')^{\frac{3}{2}}\mathcal{F}^{(\alpha\beta)(ij)}.
\end{aligned}
\tag{47}
$$

Here we have introduced the Lorentz generators $\sigma_{\mu\nu}=\frac{1}{4}\left(\sigma_\mu\bar{\sigma}_\nu-\sigma^\nu\bar{\sigma}_\mu\right)$. The world sheet integral I is given in (42). After all, this amplitude is evaluated as

$$
\begin{aligned}
&\langle\!\langle V_A^{(0)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle_{\mathrm{(S,S)}}\\
&=\frac{4\pi^2 i}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}p_{1\mu}A_\nu(p_1)\overline{\Lambda}_{\dot{\alpha}i}(p_2)\overline{\Lambda}^{\dot{\alpha}}{}_j(p_3)\right](2\pi\alpha')^{\frac{3}{2}}e^{-\frac{1}{4}\pi i}\mathcal{F}^{(\alpha\beta)(ij)}.
\end{aligned}
\tag{48}
$$

The auxiliary field insertion $\langle\!\langle V_H V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}}\rangle\!\rangle$ also contributes. The calculation of this amplitude is essentially the same as (47). The result is

$$
\begin{aligned}
&\langle\!\langle V_H^{(0)}(p_1)V_{\overline{\Lambda}}^{(-1/2)}(p_2)V_{\overline{\Lambda}}^{(-1/2)}(p_3)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle_{\mathrm{(S,S)}}\\
&\qquad=\frac{1}{4}\frac{1}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[(\sigma^{\mu\nu})_{\alpha\beta}H_{\mu\nu}(p_1)\overline{\Lambda}_{\dot{\alpha}i}(p_1)\overline{\Lambda}_j^{\dot{\alpha}}(p_3)\right]C^{(\alpha\beta)(ij)}.
\end{aligned}
\tag{49}
$$

By adding another color ordered amplitude and making the phase shift of $\overline{\Lambda}$, we obtain the graviphoton induced Lagrangian

$$
\mathcal{L}\;=\;=-\frac{i}{2}\frac{1}{kg_{\mathrm{YM}}^2}\mathrm{tr}\left[\left\{(\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x))-\frac{i}{2}H_{\mu\nu}(x)\right\}\overline{\Lambda}_{\dot{\alpha}i}(x)\overline{\Lambda}^{\dot{\alpha}}{}_j(x)\right]C^{\mu\nu(ij)}. \tag{50}
$$

Here we have defined $C^{\mu\nu(ij)}\equiv(\sigma^{\mu\nu})_{\alpha\beta}C^{(\alpha\beta)(ij)}$.

- $\underline{\langle\!\langle V_{\bar{\varphi}}V_{\overline{\Lambda}}V_{\overline{\Lambda}}V_{\bar{\varphi}}V_{\mathcal{F}}\rangle\!\rangle}$

The amplitude of the form $\langle\!\langle V_{\bar{\varphi}}V_{\overline{\Lambda}}V_{\overline{\Lambda}}V_{\bar{\varphi}}V_{\mathcal{F}}\rangle\!\rangle$ is also the candidate for the non-vanishing amplitude:

$$
\begin{aligned}
&\langle\!\langle V_{\bar{\varphi}}^{(0)}(p_1)V_{\Lambda}^{(-1/2)}(p_2)V_{\Lambda}^{(-1/2)}(p_3)V_{\bar{\varphi}}^{(0)}(p_4)V_{\mathcal{F}}^{(-1/2,-1/2)}\rangle\!\rangle\\
&=\frac{1}{2\pi^2\alpha'^2}\frac{1}{kg_{\mathrm{YM}}^2}(2\pi\alpha')^{\frac{7}{2}}(2i)^2\mathrm{tr}\left[\bar{\varphi}(p_1)\Lambda^{k\gamma}(p_2)\Lambda^{l\delta}(p_3)\bar{\varphi}(p_4)\right]\mathcal{F}^{\alpha\beta ij}
\end{aligned}
$$

$$\times \int \frac{\prod_j dy_j}{dV_{\rm CKG}} \langle e^{-\frac{1}{2}\phi(y_2)} e^{-\frac{1}{2}\phi(y_3)} e^{-\frac{1}{2}\phi(z)} e^{-\frac{1}{2}\phi(\bar{z})} \rangle \langle S_k(y_2) S_l(y_3) S_i(z) S_j(\bar{z}) \rangle$$
$$\times \Big\langle \left(\partial \overline{Z}(y_1) + i(2\pi\alpha')^{\frac{1}{2}} p_1 \cdot \psi \overline{\Psi}(y_1) \right) S^{(-)}(y_2) S^{(-)}(y_3) S_\gamma(y_2) S_\delta(y_3)$$
$$\times \left(\partial \overline{Z}(y_4) + i(2\pi\alpha')^{\frac{1}{2}} p_4 \cdot \psi \overline{\Psi}(y_4) \right) S^{(-)}(z) S^{(-)}(\bar{z}) S_\alpha(z) S_\beta(\bar{z}) \Big\rangle \left\langle \prod_{j=1}^{4} e^{i\sqrt{2\pi\alpha'} p_j \cdot X(y_j)} \right\rangle . \tag{51}$$

The $\partial\overline{Z}\partial\overline{Z}$ part and the cross terms does not contribute to the amplitude because the ϕ_3-charge can not be canceled. The non-zero contribution comes from the $p_1 \cdot \psi\overline{\Psi}\ p_2 \cdot \psi\overline{\Psi}$ part only. The correlator is reduced to the form

$$\begin{aligned} & i^2(2\pi\alpha') p_{1\mu} p_{2\nu} \langle \psi^\mu(y_1) S_\gamma(y_2) S_\delta(y_3) \psi^\nu(y_4) S_\alpha(z) S_\beta(\bar{z}) \rangle \\ & \quad \times \langle \overline{\Psi}(y_1) S^{(-)}(y_2) S^{(-)}(y_3) \overline{\Psi}(y_4) S^{(-)}(z) S^{(-)}(\bar{z}) \rangle \\ & \quad \times \langle e^{-\frac{1}{2}\phi(y_2)} e^{-\frac{1}{2}\phi(y_3)} e^{-\frac{1}{2}\phi(z)} e^{-\frac{1}{2}\phi(\bar{z})} \rangle . \end{aligned} \tag{52}$$

However this is higher α' order contribution so that it does not survive in the zero-slope limit in our scaling.

- $\underline{\langle\!\langle V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\!\rangle}$

The amplitude is

$$\begin{aligned} & \langle\!\langle V_{\overline{\Lambda}}^{(-1/2)}(p_1) V_{\overline{\Lambda}}^{(-1/2)}(p_2) V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle_{(S,S)} \\ & = \frac{1}{2\pi^2\alpha'^2} \frac{1}{k g_{\rm YM}^2} (2\pi\alpha')^{\frac{5}{2}} {\rm tr}\left[\overline{\Lambda}_{\dot{\alpha}k}(p_1) \overline{\Lambda}_{\dot{\beta}l}(p_2) \right] \mathcal{F}^{\alpha\beta ij} \\ & \times \int \frac{\prod_j dy_j}{dV_{\rm CKG}} \langle e^{-\frac{1}{2}\phi(y_1)} e^{-\frac{1}{2}\phi(y_2)} e^{-\frac{1}{2}}\phi(z) e^{-\frac{1}{2}\phi(\bar{z})} \rangle \langle S^{\dot{\alpha}}(y_1) S^{\dot{\beta}}(y_2) S_\alpha(z) S_\beta(\bar{z}) \rangle \\ & \times \langle S^{(+)}(y_1) S^{(+)}(y_2) S^{(-)}(z) S^{(-)}(\bar{z}) \rangle \langle S^k(y_1) S^l(y_2) S_i(z) S_j(\bar{z}) \rangle \left\langle \prod_{j=1}^{3} e^{i\sqrt{2\pi\alpha'} p_j \cdot X(y_j)} \right\rangle . \end{aligned} \tag{53}$$

The effective rule for the four-dimensional spin field correlator is

$$\langle S^{\dot{\alpha}}(y_1) S^{\dot{\beta}}(y_2) S_\alpha(z) S_\beta(\bar{z}) \rangle = \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon_{\alpha\beta} (y_1 - y_2)^{-\frac{1}{2}} (z - \bar{z})^{-\frac{1}{2}}, \tag{54}$$

which gives vanishing contribution when contracted with the (S,S) type of graviphoton.

- $\underline{\langle\!\langle V_{H_{\varphi\bar{\varphi}}} V_{\overline{\Lambda}} V_{\overline{\Lambda}} V_{\mathcal{F}} \rangle\!\rangle}$

The amplitude is

$$\begin{aligned}
&\langle\!\langle V^{(0)}_{H_{\varphi\bar{\varphi}}}(p_1) V^{(-1/2)}_{\overline{\Lambda}}(p_2) V^{(-1/2)}_{\overline{\Lambda}}(p_3) V^{(-1/2,-1/2)}_{\mathcal{F}} \rangle\!\rangle \\
&= \frac{1}{2\pi^2\alpha'^2}\frac{1}{kg^2_{\rm YM}}(2\pi\alpha')^{\frac{7}{2}}(-i\sqrt{2})\mathrm{tr}\left[H_{\varphi\bar{\varphi}}(p_1)\overline{\Lambda}_{\dot{\gamma}k}(p_2)\overline{\Lambda}_{\dot{\delta}l}(p_3)\right]\mathcal{F}^{\alpha\beta ij} \\
&\times \int \frac{\prod_j dy_j}{dV_{\rm CKG}}\langle e^{-\frac{1}{2}\phi(y_2)}e^{-\frac{1}{2}\phi(y_3)}e^{-\frac{1}{2}\phi(z)}e^{-\frac{1}{2}\phi(\bar{z})}\rangle \\
&\times \langle S^k(y_2)S^l(y_3)S_i(z)S_j(\bar{z})\rangle\langle S^{\dot{\gamma}}(y_2)S^{\dot{\delta}}(\bar{z})\rangle\langle S_\alpha(z)S_\beta(\bar{z})\rangle \\
&\times \langle \Psi\overline{\Psi}(y_1)\ S^{(+)}(y_2)S^{(+)}(y_3)S^{(-)}(z)S^{(-)}(\bar{z})\rangle \left\langle \prod_{j=1}^{3} e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle.
\end{aligned} \tag{55}$$

In this case, there is a factor $\varepsilon_{\alpha\beta}$ coming from the spin field correlator $\langle S_\alpha(z)S_\beta(\bar{z})\rangle$. Thus when it is contracted with the (S,S) type of the graviphoton, this part gives vanishing result.

Altogether, the interaction term $\mathcal{L}_{(S,S)}$ in the Lagrangian induced by the (S,S) type of the graviphoton field strength at the lowest order is

$$\begin{aligned}
\mathcal{L}_{(S,S)} = &-\frac{1}{\sqrt{2}}\frac{1}{kg^2_{\rm YM}}\mathrm{tr}\left[C^{(\alpha\beta)(ij)}\left\{\partial_\mu\bar{\varphi}(x)+H_{A\bar{\varphi}_\mu}(x),(\sigma^\mu)_{\alpha\dot{\alpha}}\overline{\Lambda}^{\dot{\alpha}}{}_i(x)\right\}\Lambda_{\beta j}(x)\right] \\
&-\frac{i}{2}\frac{1}{kg^2_{\rm YM}}\mathrm{tr}\left[\left\{(\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x))-\frac{i}{2}H_{\mu\nu}(x)\right\}\overline{\Lambda}_{\dot{\alpha}i}(x)\overline{\Lambda}^{\dot{\alpha}}{}_j(x)\right]C^{\mu\nu(ij)}.
\end{aligned} \tag{56}$$

After integrating out the auxiliary fields, we find effective interaction terms are written as

$$\begin{aligned}
\mathcal{L}_{(S,S)} \;=\; &-\frac{1}{\sqrt{2}}\frac{1}{g^2_{\rm YM}}\frac{1}{k}\mathrm{tr}\left[C^{(\alpha\beta)(ij)}\{D_\mu\bar{\varphi},(\sigma^\mu)_{\alpha\dot{\alpha}}\overline{\Lambda}^{\dot{\alpha}}{}_i\}\Lambda_{\beta j}\right] \\
&-\frac{i}{2}\frac{1}{g^2_{\rm YM}}\frac{1}{k}\mathrm{tr}\left[F_{\mu\nu}\overline{\Lambda}_i\overline{\Lambda}_j C^{\mu\nu(ij)}\right]+\frac{1}{8}\frac{1}{g^2_{\rm YM}}\frac{1}{k}\mathrm{tr}\left[\overline{\Lambda}_i\overline{\Lambda}_j C^{\mu\nu(ij)}\overline{\Lambda}_k\overline{\Lambda}_l C_{\mu\nu}{}^{(kl)}\right].
\end{aligned} \tag{57}$$

If we consider the case that only the part $C^{\alpha\beta} \equiv C^{\alpha\beta 11}$ is non-zero then we find the deformed Lagrangian

$$\mathcal{L}_c = \mathcal{L}^{\mathcal{N}=2}_{\rm SYM} + \mathcal{L}_{(S,S)} \tag{58}$$

precisely coincides with the one constructed in the $\mathcal{N}=1/2$ superspace with the Moyal product [15][1]

$$\begin{aligned}\mathcal{L} &= \frac{1}{g_{\rm YM}^2}\frac{1}{k}{\rm tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}-i\bar{\lambda}_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha}D_\mu\lambda_\alpha-i\bar\psi_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha}D_\mu\psi_\alpha\\ &\qquad -(D_\mu\bar{A})(D^\mu A)-i\sqrt{2}[\bar{A},\psi^\mu]\lambda_\alpha-i\sqrt{2}[A,\bar\psi_{\dot\alpha}]\bar\lambda^{\dot\alpha}-\frac{1}{2}[A,\bar{A}]^2\Big]\\ &\quad +\frac{1}{g_{\rm YM}^2}\frac{1}{k}{\rm tr}\Big[-\frac{i}{2}C^{\mu\nu}F_{\mu\nu}\bar\lambda_{\dot\alpha}\bar\lambda^{\dot\alpha}+\frac{1}{8}|C|^2(\bar\lambda_{\dot\alpha}\bar\lambda^{\dot\alpha})^2-\frac{\sqrt{2}}{2}C^{\alpha\beta}\{D_\mu\bar{A},(\sigma^\mu)_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}\}\psi_\beta\Big].\end{aligned} \tag{59}$$

Here, we have defined $\bar\varphi=\bar{A}$, $\Lambda^1\equiv\lambda,\overline{\Lambda}_1\equiv\bar\lambda,\Lambda_1\equiv\psi,\overline{\Lambda}^1=\bar\psi$. Actually, this (S,S) type of the R-R background $\mathcal{F}^{(\alpha\beta)(11)}$ corresponds to the graviphoton vertex operator which induces the non-anticommutativity in the $\mathcal{N}=1$ superspace [7],

$$V_{\mathcal{F}}^{(-1/2,-1/2)}(z,\bar z)=(2\pi\alpha')\mathcal{F}^{\alpha\beta(11)}\left[S_\alpha(z)S^{(---)}e^{-\frac{1}{2}\phi(z)}S_\beta(\bar z)S^{(---)}(\bar z)e^{-\frac{1}{2}\phi(\bar z)}\right]. \tag{60}$$

It is worth to mention that deformation to the super Yang-Mills action in the background (60) terminates at the quadratic order in $\mathcal{F}$ though it is not the case for the general graviphoton background. The $\mathcal{F}^2$ term appeared in the (58) is the only possible one.

We thus conclude that the $\mathcal{N}=2$ super Yang-Mills theory defined on the $\mathcal{N}=1/2$ superspace $\{\theta^\alpha,\theta^\beta\}=C^{\alpha\beta}$ is the effective theory of the D3-branes in the background graviphoton field of the type (60). The effective Lagrangian preserves only a part of the original supersymmetry but the canonical gauge invariance is intact [15]. Note that the background corresponding to the Lagrangian (58) is self-dual and does not receive the gravitational back-reaction.

The general type of the graviphoton background $\mathcal{F}^{(\alpha\beta)(ij)}$ seems to correspond to the non-singlet deformation $\{\theta^{i\alpha},\theta^{j\beta}\}=C^{\alpha\beta}b^{ij}$ of $\mathcal{N}=2$ harmonic superspace. In fact, decomposing the scaled graviphoton field strength $C^{(\alpha\beta)(ij)}$ into the the form $C^{\alpha\beta}b^{ij}$ and setting $b^{ij}=\delta^i_1\delta^j_1$ [24]. In fact, for b^{ij} satisfying $b^{ij}b_{ij}=0$, the exact deformed $\mathcal{N}=2$ abelian gauge theory was obtained in [11], which is of the form

$$\mathcal{L} = -\partial_\mu\varphi\partial^\mu\bar\varphi-\frac{1}{4}F_{\mu\nu}(F^{\mu\nu}+\tilde{F}^{\mu\nu})-i\Lambda^{\alpha i}(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu\overline{\Lambda}^{\dot\alpha}_i$$

[1]Here, compared with the Lagrangian in [15], we have rescaled $A_\mu\to\frac{1}{g_{\rm YM}^2}A_\mu$, $(A,\bar{A})\to\frac{1}{g_{\rm YM}^2}(A,\bar{A})$, $C^{\alpha\beta}\to\frac{1}{g_{\rm YM}^2}C^{\alpha\beta}$.

$$+4\sqrt{2}iC^{\alpha\beta}b^{ij}(\sigma^{\mu})_{\alpha\dot{\alpha}}\partial_{\mu}\bar{\varphi}\cdot\overline{\Lambda}^{\dot{\alpha}}{}_{i}\Lambda_{\beta j}-2C^{\mu\nu}b^{ij}F_{\mu\nu}\overline{\Lambda}_{\dot{\alpha}i}\overline{\Lambda}^{\dot{\alpha}}{}_{j}-2C^{\mu\nu}b^{ij}C_{\mu\nu}b^{kl}(\overline{\Lambda}_{i}\overline{\Lambda}_{j})(\overline{\Lambda}_{k}\overline{\Lambda}_{l}). \tag{61}$$

If we identify $C^{\alpha\beta}b^{ij}\equiv\frac{i}{4}C^{(\alpha\beta)(ij)}$, the non-singlet deformed Lagrangian (61) exactly agrees with the (S,S) type deformed theory (58). For the non-singlet case with $b^{ij}b_{ij}\neq 0$, we can show that the deformed Lagrangian (58) agrees with that of [11] at the first order in bc.

4 Conclusions and Discussion

In this paper, we have written down the low-energy effective Lagrangian of $\mathcal{N}=2$ supersymmetric gauge theory from the open superstring amplitudes in the graviphoton background. The structure of the deformed action depends on the scaling condition of the background in the zero-slope limit. We have chosen that the deformation parameter have the same dimension of the non-anticommutativity parameter of the superspace, *i.e.* the graviphoton polarization scales as $(2\pi\alpha')^{\frac{3}{2}}\mathcal{F}=$ fixed in the zero-slope limit.

Compared with the deformation of $\mathcal{N}=1$ super Yang-Mills theory [7], where only finite number of graviphoton vertex operator insertion in the disk amplitude is allowed, arbitrary number of graviphoton vertex operators can be inserted in the disk amplitudes in the case of the deformation of $\mathcal{N}=2$ theory.

We have discussed that the graviphoton field strength $\mathcal{F}^{\alpha\beta ij}$ can be classified into four types: (S,S), (S,A), (A,S) and (A,A) types. In the present work, we have investigated the (S,S) type deformation in detail. For (S,S) type deformation, we have shown that the $\mathcal{N}=2$ super Yang-Mills theory defined on $\mathcal{N}=1$ non(anti)commutative superspace is precisely equivalent to the effective theory of the D3-branes in the presence of the self-dual $\mathcal{F}^{(\alpha\beta)(11)}$ graviphoton background. We also find that the deformed $\mathcal{N}=2$ abelian gauge theory defined on non(anti)commutative harmonic superspace $\{\theta^{i\alpha},\theta^{j\beta}\}=C^{\alpha\beta}b^{ij}$ with $b^{ij}b_{ij}=0$ [11] is reproduced by the disk amplitude. On the other hand the deformations with parameters $\mathcal{F}^{[\alpha\beta](ij)}$ and $\mathcal{F}^{(\alpha\beta)[ij]}$ do not correspond to the deformation of superspace. These types of background would give new types of deformation of $\mathcal{N}=2$ theory. In [14], the (S,A) type of the graviphoton $\mathcal{F}^{(\alpha\beta)[ij]}$ has been discussed. They considered the scaling $(2\pi\alpha')^{\frac{1}{2}}\mathcal{F}=$ fix, which is different from ours. In this scaling, the non-zero contribution comes from only two amplitudes

$$\langle\langle V_A V_{\bar{\varphi}} V_{\mathcal{F}} \rangle\rangle, \qquad \langle\langle V_H V_{\bar{\varphi}} V_{\mathcal{F}} \rangle\rangle. \tag{62}$$

Thus the effective Lagrangian after integrating out the auxiliary field is shown to be

$$\mathcal{L}_c = \mathcal{L}_{\mathrm{SYM}}^{\mathcal{N}=2} + \mathcal{L}', \tag{63}$$

where the induced interaction $\mathcal{L}'$ is simply given by

$$\mathcal{L}' = \frac{1}{kg_{\mathrm{YM}}^2} \mathrm{tr} \left[F_{\mu\nu} \bar{\varphi} \tilde{C}^{\mu\nu} + (\bar{\varphi} \tilde{C}^{\mu\nu})^2 \right]. \tag{64}$$

We can also study the (A, A) type deformation, which is expected to correspond to the singlet deformation of $\mathcal{N} = 2$ harmonic superspace [10]. But it is easily found that this theory includes divergence such as $\frac{1}{(2\pi\alpha')^2} \mathrm{tr} \bar{\varphi} C$ in the zero-slope limit. Therefore it is necessary to consider renormalization or back reactions in this type of deformation. We will examine deformed $\mathcal{N} = 2$ gauge theories corresponding to these types of graviphoton backgrounds in a forthcoming paper. We can also extend the present construction to deformed $\mathcal{N} = 4$ supersymmetric gauge theory, where the deformed Lagrangian in the $\mathcal{N} = 1/2$ superspace is known [26]. String theory calculation would provide more general type deformation of $\mathcal{N} = 4$ theory. This subject will be also discussed in a separate paper.

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A Classification of possible disk amplitudes with one graviphoton insertion

All possible contributions from the disk amplitudes including one graviphoton vertex operator are summarized as follows:

$$\text{(I).}\quad \langle\!\langle V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(II).}\quad \langle\!\langle V_{A}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \langle\!\langle V_{H}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\begin{aligned}\text{(III).}\quad & \langle\!\langle V_{A}^{(0)} V_{A}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \langle\!\langle V_{A}^{(0)} V_{H}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \\ & \langle\!\langle V_{H}^{(0)} V_{A}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle \ , \langle\!\langle V_{H}^{(0)} V_{H}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle\end{aligned}$$

$$\begin{aligned}\text{(IV).}\quad & \langle\!\langle V_{\varphi}^{(0)} V_{\bar{\varphi}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \ \langle\!\langle V_{H_{A\varphi}}^{(0)} V_{\bar{\varphi}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \\ & \langle\!\langle V_{\varphi}^{(0)} V_{H_{A\bar{\varphi}}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \ \langle\!\langle V_{H_{A\varphi}}^{(0)} V_{H_{A\bar{\varphi}}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle\end{aligned}$$

$$\text{(V).}\quad \langle\!\langle V_{H_{\varphi\bar{\varphi}}}^{(0)} V_{H_{\varphi\bar{\varphi}}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(VI).}\quad \langle\!\langle V_{A}^{(0)} V_{H_{\varphi\bar{\varphi}}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \ \langle\!\langle V_{H}^{(0)} V_{H_{\varphi\bar{\varphi}}}^{(0)} V_{\bar{\varphi}}^{(-1)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(VII).}\quad \langle\!\langle V_{\Lambda}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\bar{\varphi}}^{(0)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \ \langle\!\langle V_{\Lambda}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{H_{A\bar{\varphi}}}^{(0)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(VIII).}\quad \langle\!\langle V_{\bar{\varphi}}^{(0)} V_{\Lambda}^{(-1/2)} V_{\Lambda}^{(-1/2)} V_{\bar{\varphi}}^{(0)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(IX).}\quad \langle\!\langle V_{\bar{\Lambda}}^{(-1/2)} V_{\bar{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(X).}\quad \langle\!\langle V_{A}^{(0)} V_{\overline{\Lambda}}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle, \ \langle\!\langle V_{H}^{(0)} V_{\overline{\Lambda}}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle$$

$$\text{(XI).}\quad \langle\!\langle V_{H_{\varphi\bar{\varphi}}}^{(0)} V_{\overline{\Lambda}}^{(-1/2)} V_{\overline{\Lambda}}^{(-1/2)} V_{\mathcal{F}}^{(-1/2,-1/2)} \rangle\!\rangle.$$

B Effective rules

The ten-dimensional correlator is computed by decomposing it into the four-dimensional part and the internal part [7]. The effective rules that enable one to calculate each part separately, is derived by the general formula [17]. For the four-dimensional spin field, we have

$$\langle S_\alpha(z) S_\beta(\bar{z}) \rangle = \varepsilon_{\alpha\beta}(z-\bar{z})^{-\frac{1}{2}}, \tag{65}$$

$$\langle S^{\dot{\alpha}}(y_1) S^{\dot{\beta}}(y_2) \rangle = \varepsilon^{\dot{\alpha}\dot{\beta}}(y_1-y_2)^{-\frac{1}{2}}, \tag{66}$$

$$\langle S^{\dot{\alpha}}(y_1) S^{\dot{\beta}}(y_2) S_\alpha(z) S_\beta(\bar{z}) \rangle = \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon_{\alpha\beta}(y_1-y_2)^{-\frac{1}{2}}(z-\bar{z})^{-\frac{1}{2}}. \tag{67}$$

If there are world sheet fermion inside the correlator, it should be carefully computed by evaluating the cocycle factor. Then, we find

$$\langle S^{\dot{\alpha}}(y_1)\psi^{\mu}(y_2)S_{\alpha}(y_3)\rangle = -\frac{1}{\sqrt{2}}(\bar{\sigma}^{\mu})^{\dot{\alpha}\beta}\varepsilon_{\beta\alpha}(y_1-y_2)^{-\frac{1}{2}}(y_2-y_3)^{-\frac{1}{2}}. \tag{68}$$

The internal part is also computed to be

$$\langle S_i(z)S_j(\bar{z})\rangle = \varepsilon_{ij}(z-\bar{z})^{-\frac{1}{2}}, \tag{69}$$

$$\begin{aligned}\langle S^i(y_1)S^j(y_2)S^k(z)S^l(\bar{z})\rangle &= [(y_1-y_2)(y_1-z)(y_1-\bar{z})(y_2-z)(y_2-\bar{z})(z-\bar{z})]^{-\frac{1}{2}} \\ &\quad\times\left[\varepsilon^{il}\varepsilon^{jk}(y_1-z)(y_2-\bar{z})-\varepsilon^{ik}\varepsilon^{jl}(y_2-z)(y_1-\bar{z})\right] \\ &= [(y_1-y_2)(y_1-z)(y_1-\bar{z})(y_2-z)(y_2-\bar{z})(z-\bar{z})]^{-\frac{1}{2}} \\ &\quad\times\left[-\varepsilon^{ij}\varepsilon^{kl}(y_1-\bar{z})(y_2-z)+\varepsilon^{il}\varepsilon^{jk}(y_1-y_2)(z-\bar{z})\right].\end{aligned} \tag{70}$$

If there are Lorentz generators in the correlator, one first should reduce it to the one which does not contain any Lorentz generator by the formula presented in [17]:

$$\begin{aligned}&\left\langle O^{(p)}(z_p)\dots O^{(j+1)}(z_{j+1}):\psi^M\psi^N(z):O^{(j)}(z_j)\cdots O^{(1)}(z_1)\right\rangle \\ &=\sum_l\left\{(M^{MN})^l{}_{l'}(z-z_l)^{-1}+(M'^{MN})^l{}_{l'}(z-z_l)^{-2}\right\} \\ &\quad\times\left\langle O^{(p)}(z_p)\dots O^{(j+1)}(z_{j+1})O^{(l')}(z_l)\cdots O^{(1)}(z_1)\right\rangle.\end{aligned} \tag{71}$$

Here, M,N is the ten-dimensional space-time indices and the matrices $(M^{MN})^l{}_{l'}$ and $(M'^{MN})^l{}_{l'}$ are specified by the OPE

$$:\psi^M\psi^N(z):O^{(l)}(w)\sim\left[(z-w)^{-2}(M'^{MN})^l{}_{l'}+(z-w)^{-1}(M^{MN})^l{}_{l'}\right]O^{(l')}(w). \tag{72}$$

The space-time Lorentz generator $:\psi^\mu\psi^\nu:$ correlates only with the four-dimensional part, so we can find

$$\langle S^{\dot{\alpha}}(z_1)\psi^\mu\psi^\nu(z_2)S^{\dot{\beta}}(z_3)\rangle = -\frac{1}{2}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}(z_1-z_3)^{\frac{1}{2}}(z_1-z_2)^{-1}(z_2-z_3)^{-1}. \tag{73}$$

If there are two Lorentz generators in the correlator, we see

$$\langle\psi^\mu\psi^\nu(y_1)\psi^\rho\psi^\sigma(y_2)S_\alpha(z)S_\beta(\bar{z})\rangle$$

$$
\begin{aligned}
= \quad & -\varepsilon_{\alpha\beta}\left(\delta^{\mu\rho}\delta^{\nu\sigma}-\delta^{\mu\sigma}\delta^{\nu\rho}\right)(y_1-y_2)^{-2}(z-\bar{z})^{-\frac{1}{2}} \\
& -\frac{1}{4}(\sigma^{\mu\nu})_\alpha{}^\gamma(\sigma^{\rho\sigma})_{\gamma\beta}(y_1-z)^{-1}(y_2-z)^{-1}(y_2-\bar{z})^{-1}(z-\bar{z})^{\frac{1}{2}} \\
& -\frac{1}{4}(\sigma^{\mu\nu})_\beta{}^\gamma(\sigma^{\rho\sigma})_{\gamma\alpha}(y_1-\bar{z})^{-1}(y_2-z)^{-1}(y_2-\bar{z})^{-1}(z-\bar{z})^{\frac{1}{2}}. \qquad (74)
\end{aligned}
$$

In the same way, we find

$$
\begin{aligned}
& \langle\psi^\mu\psi^\nu(y_1)S^{\dot{\alpha}}(y_2)S^{\dot{\beta}}(y_3)S_\alpha(z)S_\beta(\bar{z})\rangle \\
&= \frac{1}{2}(y_2-y_3)^{-\frac{1}{2}}(z-\bar{z})^{-\frac{1}{2}}\left[(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}\frac{(y_2-y_3)}{(y_1-y_2)(y_1-y_3)}+(\sigma^{\mu\nu})_{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{(z-\bar{z})}{(y_1-z)(y_1-\bar{z})}\right]. \qquad (75)
\end{aligned}
$$

In the case of the internal "Lorentz generator" $:\Psi\overline{\Psi}:$, the same formula can be used. The result is

$$
\langle:\Psi\overline{\Psi}(y_1):\overline{\Psi}(y_2)\Psi(y_3)\rangle=\frac{1}{(y_1-y_2)(y_1-y_3)}. \qquad (76)
$$

From the ten-dimensional calculation, we find

$$
\begin{aligned}
& \langle S_\gamma(y_1)S^{\dot{\delta}}(y_2)\psi^\mu(y_3)S_\alpha(z)S_\beta(\bar{z})\rangle \\
&= \frac{1}{\sqrt{2}}(y_1-z)^{-\frac{1}{2}}(y_1-\bar{z})^{-\frac{1}{2}}(y_2-y_3)^{-\frac{1}{2}}(y_1-y_3)^{\frac{1}{2}}(y_3-z)^{\frac{1}{2}}(y_3-\bar{z})^{\frac{1}{2}}(z-\bar{z})^{-\frac{1}{2}} \\
&\quad \times\left[(\sigma^\mu)_\gamma{}^{\dot{\delta}}\varepsilon_{\alpha\beta}(y_1-y_3)^{-1}+(\sigma^\mu)_\alpha{}^{\dot{\delta}}\varepsilon_{\gamma\beta}(y_3-z)^{-1}-(\sigma^\mu)_\beta{}^{\dot{\delta}}\varepsilon_{\gamma\beta}(y_3-\bar{z})^{-1}\right]. \qquad (77)
\end{aligned}
$$

For X^μ fields, the general formula [17] is useful:

$$
\left\langle\prod_{\{l\}}\varepsilon^{(l)}\cdot\partial X(z_l)\prod_j e^{i\sqrt{2\pi\alpha'}p^{(j)}\cdot X(z_j)}\right\rangle=\prod_{\{l\}}\left[\frac{\partial}{\partial z_l}\bigg|_{\sqrt{2\pi\alpha'}p^{(l)}\to\varepsilon^{(l)}}\right]\prod_{i>j}(z_i-z_j)^{2\pi\alpha' p^{(i)}\cdot p^{(j)}}. \qquad (78)
$$

By using this formula, we find

$$
\left\langle A_\mu(p_1)\partial X^\mu(y_1)\prod_{j=1}^n e^{i\sqrt{2\pi\alpha'}p_j\cdot X(y_j)}\right\rangle=\prod_{i<j}(y_i-y_j)^{2\pi\alpha' p_i\cdot p_j}\times i(2\pi\alpha')^{\frac{1}{2}}\sum_{j=2}^n\left[\frac{A_\mu(p_1)p_j^\mu}{y_1-y_j}\right], \qquad (79)
$$

and

$$
\begin{aligned}
& \left\langle A_\mu(p_1)\partial X^\mu(y_1)e^{i\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}A_\nu(p_2)\partial X^\nu(y_2)e^{i\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}e^{i\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\right\rangle \\
&= (y_1-y_2)^{2\pi\alpha' p_1\cdot p_2}(y_1-y_3)^{2\pi\alpha' p_1\cdot p_3}(y_2-y_3)^{2\pi\alpha' p_2\cdot p_3} \\
&\times\left[\frac{A_\mu(p_1)A^\mu(p_2)}{(y_1-y_2)^2}+\frac{(2\pi\alpha')A_\mu(p_1)p_2^\mu A_\nu(p_2)p_1^\nu}{(y_1-y_2)^2}+\frac{(2\pi\alpha')A_\mu(p_1)p_3^\mu A_\nu(p_2)p_1^\nu}{(y_1-y_2)(y_1-y_3)}\right. \\
&\qquad\left.-\frac{(2\pi\alpha')A_\mu(p_1)p_2^\mu A_\nu(p_2)p_3^\nu}{(y_1-y_2)(y_2-y_3)}-\frac{(2\pi\alpha')A_\mu(p_1)p_3^\mu A_\nu(p_2)p_3^\nu}{(y_1-y_3)(y_2-y_3)}\right]. \qquad (80)
\end{aligned}
$$

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